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Empirical scalings of cross-field heat diffusivities in the scrape-off layer of Alcator C-Mod from a 2-D interpretive model

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Abstract

A new 2-D interpretive code for the tokamak edge plasma, EDGEFIT, has been developed. It infers the anomalous cross-field heat diffusivity, χ, in the scrape-off layer (SOL) of Alcator C-Mod from the experimental data solving the heat conduction equation in the real magnetic geometry. Measurements of temperature from the Langmuir probes on the divertor target plates are used for setting up the boundary conditions. Cross-field diffusivities are found by matching the cross-field temperature profiles measured by a fast-scanning Langmuir probe (FSP). The separatrix values of the extracted heat diffusivity, χ_0, are statistically investigated for a large number of Alcator C-Mod discharges. No correlation between χ_0, on one hand, and plasma temperature on separatrix, magnetic field strength and the safety factor, on the other hand, has been found although parameters vary significantly in the investigated data set (20–80 eV for the separatrix temperature, 2.8–7.9 Tesla for the toroidal field, 3.0–7.4 for the safety factor). However it has been found that χ_0 values have a weak but noticeable trend to get smaller as the midplane neutral pressure, the divertor pressure and the separatrix plasma density grow. The most significant factor found to affect χ_0 is a bypass neutral gas leak existing in the Alcator C-Mod divertor. It appears that χ_0 is smaller by a factor of 3 on an average when the bypass leak is closed. This can be possibly due to a perturbation of plasma temperature at the FSP location caused by the flow of neutral gas associated with the bypass, or a systematic error in the FSP measurements.

Keywords: Edge transport; Heat diffusivity

1. Introduction

One of the most critical parameters of a fusion reactor is the power width in the edge plasma since it determines the magnitude of the heat flux density that has to be accommodated by the divertor. The power width depends on the anomalous cross-field thermal diffusivity χ in the edge plasma. This important parameter, χ, is generally believed to be about 0.1–1 m^2/s. This is what is usually cited in reports from edge plasma modeling where the anomalous transport is inferred from matching the scrape-off layer (SOL) cross-field density and temperature profiles [1]. However to design next generation fusion devices one needs to know how χ scales with other parameters of the reactor such as plasma density, temperature, magnetic field strength etc. In order to infer such scalings a systematic study has to be undertaken for a large number of discharges in broad range of parameters. Such a study is not practical using an existing tokamak edge plasma code such as UEDGE [2] which contains very detailed physics models but running it is difficult and requires much time even in modeling a single discharge. Another way to extract information about anomalous transport in the edge plasma is the so-called 'interpretive approach'. Here a simpler model is used drawing on as much data as possible directly from the experiment. For the edge plasma this interpretive approach has typically employed a simple 1D approximation, the onion-skin model [3]. The advantage is that it requires very little time to process hundreds of discharges. One can then use statistical methods to search for empirical trends in the deduced transport coefficients. However there is no rigorous justification for the onion-skin approximation

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for the 2-D geometry. To address this problem a new code, EDGEFIT, has been developed to combine the advantage of an interpretive code with an accurate 2-D treatment of the real geometry.

2. Model description

EDGEFIT solves the heat conduction equation for the SOL domain (Fig. 1) which is limited by two flux surfaces in the radial direction and the target plates in the poloidal direction and finds the optimal anomalous plasma diffusion coefficient that best matches the experimentally measured radial plasma temperature profile in SOL. The real magnetic geometry is based on EFIT reconstruction [4] produced individually for each modeled shot and time slice. For the along-field heat transport the classical electron heat conduction $K_{\|} \propto T^{2.5}$ is assumed, and for the transverse direction it is assumed that the heat flux is given by

$$q_{\perp} = -n\chi\nabla_{\perp} T, \quad (1)$$

where n is the experimental plasma density taken from the fast-scanning probe (FSP) and taken constant on the flux surface. In some regimes taking plasma pressure constant on the flux surface could be a better approximation but this would not make much difference here since the non-linear parallel heat conduction causes temperature to be essentially constant on the flux surface in SOL (far from the target plates).

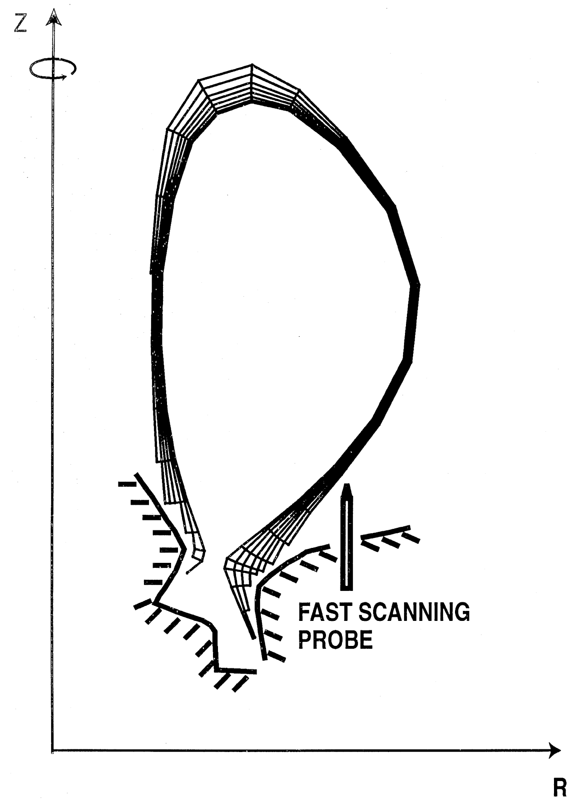

Fig. 1. Computational mesh for the SOL domain for shot 960208008, time slice 0.755 s.

An unknown effective heat diffusivity, χ, lumps together heat diffusion and convection. The source term (radiation) is completely neglected here since radiation presumably occurs mainly below the x-point level and thus should have a small effect on the upstream radial T profile in SOL. The steady state heat conduction equation solved by EDGEFIT is

$$\nabla(K_{\|}\nabla_{\|}T + n\chi\nabla_{\perp}T) = 0 \quad (2)$$

with the effective heat diffusivity χ assumed to be only a function of the flux coordinate ρ. This function $\chi(\rho)$ is represented by a few parameters that are iteratively optimized by EDGEFIT until the calculated 2-D temperature profile agrees with the measured radial temperature profile as given by the FSP. To set up correctly the boundary problem for Eq. (2) one needs to specify the boundary conditions for the computational domain. On the target plate boundary a fixed temperature profile is set according to the divertor probe data for given shot and time slice. On the outer boundary the heat flux is set to be zero. On the inner boundary the poloidal profile of temperature is fixed assuming (similarly to the onion-skin model) that the temperature-like variable $T^{3.5}$ is a quadratic function of the length of the magnetic line passing through three temperature data points on this magnetic line: one from the FSP and two from the divertor probes on the outer and inner sides. It has been verified in our simulations that χ_0 depends only slightly on whether this boundary condition is set up on the inner boundary, or just a constant value of T, as given by the FSP, is fixed on this boundary. This lends confidence that the extracted χ_0 is not very sensitive to the details of this boundary condition. The model for the cross-field transport of thermal energy (Eq. (1)) may be incorrect due to the convective component in the energy transport [5]. Then the inferred values of χ have the meaning of the 'effective' thermal diffusivity. If the cross-field thermal energy flux has a convective term in it

$$q_{\perp} = -n\chi\nabla_{\perp}T - 2.5\,TD_{\perp}\nabla_{\perp}n, \quad (3)$$

then the effective χ can include both heat diffusion and convection

$$\chi^{\rm ef} = \chi + 2.5\,D_{\perp}{\rm d}\,[\ln(n)]/{\rm d}[\ln(T)]. \quad (4)$$

Similarly the effective χ can include a pinch term.

3. Computational approach

The direct problem of solving the non-linear steady state heat conduction equation for given radial thermal diffusivity profile $\chi(\rho)$ is performed by the Newton iteration [6]. Here ρ is the radial coordinate labeling flux surfaces by their distance from the separatrix at the outer midplane. In an external optimization loop, $\chi(\rho)$ represented by a piece-wise linear function with several free parameters is optimized to match the data. This solves the inverse problem of finding $\chi(\rho)$ for given temperature data. This external optimization is made by the downhill simplex method [6]. The optimization is completely automatic and takes just a few minutes on a VAX/VMS workstation. Hundreds of discharges have been analyzed by EDGEFIT, a task that would be impractical using a large predictive code like UEDGE.

4. Simulations and sensitivity studies

As expected, the calculated radial T profile is steeper with small χ and flatter for large χ (Fig. 2). In general a match with experimental Alcator C-Mod radial T profiles is obtained for χ in the range 0.1–10 m^2/s. Solving the direct problem of finding a 2-D profile of $T(\rho,\theta)$ for a given $\chi(\rho)$ profile performs a mapping from the χ space into the T space. The approximate inverse mapping is performed by finding the best matching $\chi(\rho)$ profile for a given radial profile $T(\rho,\theta_0)$ corresponding to θ_0, the poloidal location of the FSP providing experimental radial T profiles. An important property found for these mappings is that two very different χ profiles can correspond to close T profiles. The T profile is relatively sensitive to the separatrix value of χ but not to the χ values further out (Fig. 2). The main reason for this is that plasma density profile is rapidly decreasing in the radial direction and for a given density profile the product $n\chi$ that enters Eq. (2) depends mainly on the separatrix value of χ. This property makes it difficult, if not impossible, to extract the whole radial profile $\chi(\rho)$ from real noisy radial T profiles. It appears that it is only the separatrix value of χ, χ_0, that can be found more or less reliably and therefore only it is discussed further. This makes the present analysis different from the onion-skin model treatment [3] where χ values for all radial positions were analyzed. In fact the onion-skin model and EDGEFIT give quite close results for the separatrix value of χ.

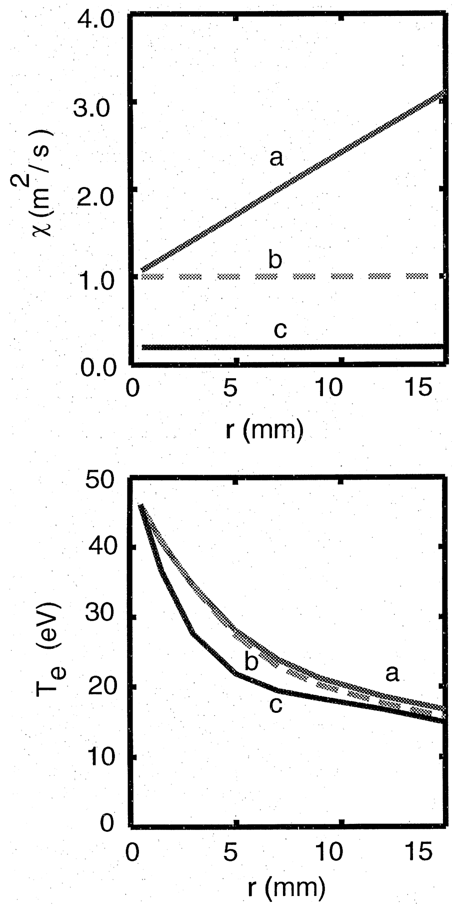

Fig. 2. Three radial χ profiles and three corresponding T profiles calculated by EDGEFIT for a particular C-Mod shot magnetic geometry with same boundary conditions. The T profiles are sensitive only to the separatrix ($r = 0$) value of c.

5. Data analysis

EDGEFIT was run on a large number of Alcator C-Mod discharges and calculated χ_0 values were placed into our database containing various information on each of the discharges. Regression analysis with statistical tests of relevance and redundancy of regressors were used to find what local (near separatrix) or global parameters could be responsible for variations of χ_0. The database contains more than 500 time slices, mostly ohmic-heated L-modes. The range of the core plasma density in the database is $0.5–2.8 \times 10^{20}$ m^{-3} , the plasma current 0.4–1.1 MA, toroidal magnetic field 2.8–7.9 T, separatrix temperature 20–80 eV, and safety factor (q_{95}) 3.0–7.4. The whole data set consists of three subsets that were treated independently. These subsets correspond to the operation periods before and after July 1995 when a bypass leak of neutral gas was partially closed in Alcator C-Mod divertor, and after December 1996 when boron was introduced in the Alcator C-Mod vessel [3]. It appears that the subsets corresponding to open and closed bypass have quite different χ_0 values. For discharges with open bypass the deduced χ_0 values are by a factor of about 3 larger on an average than for those with bypass closed. This has been previously found from the onion-skin model [3]. Modeling of Alcator C-Mod with the UEDGE code showed that a neutral gas bypass leak in the divertor can cause a local perturbation of plasma temperature in SOL. This may be a possible explanation for this effect. Another possibility is a systematic error in the FSP measurements.

Within each of the data subsets the linear regression analysis was applied in search for a power law scaling of χ_0 versus other parameters according to

$$\ln(\chi_0) = C_0 + C_1 \ln(X_1) + C_2 \ln(X_2) + \cdots + C_N \ln(X_N), \quad (5)$$

where C_i are some fitted constants and X_i are the independent variables. Various sets of plasma parameters were tested as independent variables X_i: core plasma density n_{core}, separatrix plasma density n_{sep}, separatrix plasma temperature T_{sep}, toroidal magnetic field B_{tor}, the safety factor q_{95}, the midplane neutral pressure P_{mid} and the divertor neutral pressure P_{bot}.

The partial F-test [7] was used to find relevance and redundancy of regressors. Some results of the statistical analysis are shown in Tables 1 and 2. Large value of the partial F-test in the tables (as compared to the 95% significance level which is roughly 3–6 in this case) would indicate that a particular regressor is relevant. Similar tables for χ_0 tested against different regressor sets were analyzed in the sense of backward elimination: going from the bottom to the top and eliminating those regressors that have the smallest partial F-values [7]. This procedure allows one to find a small set of regressors that describes the data almost as well as a complete model. For our data it turned out that only single regressor models could be accepted.

The values of χ_0 appeared to be correlated with the separatrix density, midplane neutral pressure, the bottom neutral pressure and with core plasma density. This can be seen from Table 1 for the case of open bypass and no boron. The plot in Fig. 3 also shows that χ_0 values have a trend to decrease as the midplane gas pressure grows. A plot of χ_0 values versus the separatrix density, core density or the bottom pressure would look similar to Fig. 3 since all these quantities are correlated with each other and with the midplane pressure, and therefore one of them can affect χ_0 through another one.

For the toroidal magnetic field B, the safety factor q_{95} and the local electron temperature T_{sep} no statistical correlation with χ_0 was found for all three data subsets. This can be seen from Table 2 for the case of open bypass and no boron. This does not necessarily imply that these parameters have no effect on χ_0 because the influence of one parameter can be compensated by another one. Still statistical methods allow one to test a particular model containing these parameters for consistency with the data. As a special case, the Bohm scaling was tested within each of the subsets. For this purpose the Bohm scaling was considered as a null hypothesis [8] to be tested against an alternative hypothesis which was taken as the best fit for χ_0 with only temperature T and magnetic field B as regressors. The F-test with the data showed that the null hypothesis can be rejected at a 95% confidence level or, equivalently, the probability of incorrectly rejecting the null hypothesis is under 5%. Thus there is a strong evidence that the Bohm scaling does not hold here.

Table 1
All subsets linear regression $Y = A_0 + \Sigma A_i X_i$

Regressors				Coefficient of determination R^2	Regression coefficients					Partial F-test for the hypothesis			
					A_0	A_1	A_2	A_3	A_4	$A_1=0$	$A_2=0$	$A_3=0$	$A_4=0$
X_1				0.1219	24.15	−0.52				26.66			
	X_2			0.1906	−0.19		−0.23				45.21		
		X_3		0.1876	0.63			−0.19				44.34	
			X_4	0.1371	33.07				−0.70				30.50
X_1	X_2			0.1921	3.82	−0.09	−0.21			0.36	16.59		
X_1		X_3		0.1878	2.14	−0.03		−0.18		0.04		15.49	
X_1			X_4	0.1419	32.27	−0.19			−0.50	1.08			4.45
	X_2	X_3		0.1951	0.16		−0.14	−0.08			1.77	1.06	
	X_2		X_4	0.1908	−2.78		−0.25		0.06		12.68		0.05
		X_3	X_4	0.1878	2.87			−0.18	−0.05			11.93	0.05
X_1	X_2	X_3		0.1953	1.65	−0.03	−0.14	−0.08		0.04	1.76	0.74	
X_1	X_2		X_4	0.1943	−3.06	−0.17	−0.25		0.22	0.83	12.36		0.53
X_1		X_3	X_4	0.1879	3.07	−0.02		−0.18	−0.03	0.01		10.75	0.01
	X_2	X_3	X_4	0.1957	−4.20		−0.15	−0.09	0.09		1.86	1.15	0.14
X_1	X_2	X_3	X_4	0.1969	−4.08	−0.11	−0.17	−0.07	0.19	0.29	2.12	0.60	0.38

Data set with no boron and open bypass. Number of samples is 194. $Y = \ln(\chi[\mathrm{m}^2/\mathrm{s}])$, $X_1 = \ln(n_{sepx}[\mathrm{M}^{-3}])$, $X_2 = \ln(P_{mid}[\mathrm{mTorr}])$, $X_3 = \ln(P_{bot}[\mathrm{mTorr}])$, $X_4 = \ln(n_{core}[\mathrm{M}^{-3}])$.

Table 2
All subsets linear regression $Y = A_0 + \Sigma A_i X_i$

Regressors				Coefficient of determination R^2	Regression coefficients					Partial F-test for the hypothesis			
					A_0	A_1	A_2	A_3	A_4	$A_1=0$	$A_2=0$	$A_3=0$	$A_4=0$
X_1				0.1219	24.15	−0.52				26.66			
	X_2			0.0178	−1.10		−0.37				3.48		
		X_3		0.0008	0.17			0.12				0.15	
			X_4	0.0011	0.67				−0.20				0.21
X_1	X_2			0.1293	22.39	−0.50	−0.24			24.46	1.62		
X_1		X_3		0.1343	27.94	−0.59		−0.51		29.45		2.72	
X_1			X_4	0.1243	24.80	−0.53			−0.29	26.88			0.52
	X_2	X_3		0.0181	−1.19		0.36	0.07			3.37	0.05	
	X_2		X_4	0.0201	−0.68		0.38		0.29		3.70		0.44
		X_3	X_4	0.0044	0.65			0.30	−0.45			0.64	0.70
X_1	X_2	X_3		0.1424	26.21	−0.57	0.25	−0.52		27.53	1.79	2.89	
X_1	X_2		X_4	0.1327	23.05	−0.51	0.26		−0.35	24.68	1.84		0.75
X_1		X_3	X_4	0.1349	28.17	−0.60		−0.59	0.19	28.65		2.31	0.13
	X_2	X_3	X_4	0.0230	−0.68		0.38	0.28	−0.52		3.62	0.58	0.97
X_1	X_2	X_3	X_4	0.1426	26.38	−0.58	0.25	−0.57	0.11	26.35	1.70	2.17	0.05

Data set with no boron and open bypass. Number of samples is 194. $Y = \ln(\chi[\mathrm{m^2/s}])$, $X_1 = \ln(n_{\mathrm{sepx}}[\mathrm{M^{-3}}])$, $X_2 = \ln(T_e[\mathrm{eV}])$, $X_3 = \ln(q_{95})$, $X_4 = \ln(B_{\mathrm{tor}}[\mathrm{Tesla}])$.

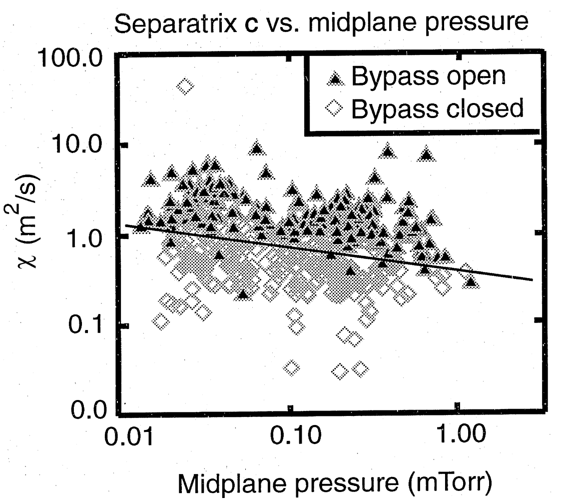

Fig. 3. Data from both boronized and non-boronized shots are shown. There is a trend for χ to decrease with growth of the midplane neutral gas pressure. The straight line is the best linear fit for the whole set of all shown data.

6. Conclusions

A 2-D interpretive model, EDGEFIT, is used to extract separatrix χ values from a large representative set of discharges of Alcator C-Mod. It is found that only the separatrix values of the effective transverse heat diffusivity χ_0 can be extracted from the real data rather than the whole radial profile of χ. The χ_0 values tend to decrease slightly with growth of the separatrix density, the midplane neutral pressure, the core plasma density and the neutral pressure in the divertor. Statistical analysis yields no correlation between χ_0 and toroidal field strength, the safety factor and the plasma temperature. The effect on χ_0 of closing of the bypass is much more pronounced than anything else. The χ_0 values appear to be smaller by a factor of about 3 on an average for discharges with the bypass closed. This effect may be caused by a local perturbation of plasma temperature at the FSP location by flows of neutral gas or, possibly, may be due to a systematic error in the FSP temperature measurements.

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References

[1] M.E. Fenstermacher et al., J. Nucl. Mater. 330–335 (1995) 220.
[2] T.D. Rognlien et al., J. Nucl. Mater. 347–351 (1992) 196.
[3] B. LaBombard et al., J. Nucl. Mater. 241–243 (1997) 149.
[4] L.L. Lao et al., Nucl. Fusion 25 (1985) 1611.
[5] M.V. Umansky et al., Phys. Plasmas 5 (1998) 3373.
[6] W.H. Press, S.T. Teukolsky, W.T. Vetterling, B.P. Flannery, Numerical Recipes in C, Cambridge University, Cambridge, 1992.
[7] B. Flury, H. Riedwyl, Multivariate Statistics: A Practical Approach, Chapman and Hall, London, 1988.
[8] B.R. Martin, Statistics for Physicists, Academic Press, London, 1971.